Bounding the top and bottom electric dipole moments from neutron experimental data

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Heavy quarks, namely, the top and bottom quarks, may show great sensitiveness to new physics effects. In particular, they might have unusually large electric dipole moments. This possibility is analyzed via the corresponding one-loop correction to the neutron electric dipole moment, d_n. The current experimental limit on d_n is used then to derive the uppers bounds $|d_t| < 3.06 \times 10^{-15}$ e-cm, $|d_b| < 1.22 \times 10^{-13}$ e-cm.

PACS numbers: 14.65.Ha, 13.40.Em,12.60.-i

The electric dipole moment (EDM) of elementary particles is a clear signal of CP violation. Even more, such an electromagnetic property would constitute itself a clear evidence of beyond-the-standard-model (SM) CP-violating effects due to the large suppression of the respective SM predictions. It is a well known fact that the only source of CP violation in the SM, namely, the Cabbibo–Kobayashi–Maskawa (CKM) phase, has an negligible effect on flavor–diagonal processes such as the EDM of elementary particles [1]. For instance, the EDM of quarks arises up to the three–loop level [2]. While the EDM of light fermions has been long studied both theoretically and experimentally (the EDM of light quarks via the neutron and proton) those of the heaviest quarks still require more attention. In fact, the top quark may be more sensitive to new sources of CP violation since it is the only known fermion with a mass of the size of the electroweak symmetry breaking scale. Indeed, most of the theories that predict new physics effects beyond the Fermi scale, predict also EDMs several orders of magnitude larger than those predicted by the SM.

In this note, we will analyze the effects induced by the EDM of the t and b quarks on the one–loop-induced EDM of the d and u quarks. We will use then the experimental limit on the neutron EDM to constrain the one associated with the t and b quarks. The EDM of heavy quarks, from now on denoted by Q, can be parametrized by the following Lagrangian

$$\mathcal{L}_{QQ\gamma} = -\frac{i}{2} d_Q \bar{Q} \gamma_5 \sigma_{\mu\nu} Q F^{\mu\nu}, \tag{1}$$

where d_Q stands for the Q quark EDM. In the unitary gauge, the contribution to the on–shell $\bar{q}q\gamma$ coupling is given through the diagram shown in Fig. 1. The respective one–loop vertex can be written as

$$\Gamma_\mu = -\frac{g^2 |V_{Qq}|^2 d_Q m_Q}{2} \int \frac{d^4k}{(2\pi)^4} \frac{P_R \gamma_\alpha [(\not{k} - \not{p}_2)\sigma_{\mu\nu} q^\nu - \sigma_{\mu\nu} q^\nu (\not{k} - \not{p}_1)] \gamma_\beta P^{\alpha\beta}}{(k^2 - m_W^2)\left((k-p_1)^2 - m_Q^2\right)\left((k-p_2)^2 - m_Q^2\right)}, \tag{2}$$

where $Q = t$ or b, $q = u$ or d, and V_{Qq} is the associated CKM element. In addition,

$$P^{\alpha\beta} \; = \; g^{\alpha\beta} - \frac{k^\alpha k^\beta}{m_W^2}. \tag{3}$$

Below we will ignore the longitudinal component of the W gauge boson owing to the fact that it contributes marginally to the above amplitude. This is a good approximation indeed as the dropped terms are proportional to increasing powers of $(m_q/m_W)^2$, which in fact is a negligible quantity. Once this approximation is taken into account, which greatly simplifies the calculation, we are left with the following term

$$\Gamma_\mu = -\frac{g^2 |V_{Qq}|^2 d_Q m_Q m_q}{2} \int \frac{d^4k}{(2\pi)^4} \frac{P_R [\sigma_{\mu\nu} q^\nu - 2i(k-p_1)_\mu]}{(k^2 - m_W^2)\left((k-p_1)^2 - m_Q^2\right)\left((k-p_2)^2 - m_Q^2\right)}. \tag{4}$$

Notice that there are contributions to both the magnetic and electric dipole moments of the q quark, but we are only interested in its CP–odd property. The contribution proportional to $p_{1\mu}$ can be expressed in terms of $\sigma_{\mu\nu} q^\nu$ via

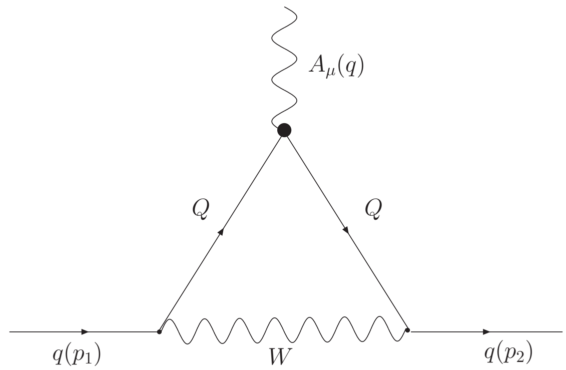

FIG. 1: Diagram contributing to the on-shell $\bar{q}q\gamma$ vertex. Q stands for a heavy quark and q for a light one.

Gordon's identity, whereas the part proportional to γ_μ can be ignored as it is proportional to m_q. Notice that gauge invariance is preserved at this order. After evaluating the integral over k, we obtain

$$d_q = \left(\frac{\alpha}{4\pi}\right)\left(\frac{|V_{Qq}|^2}{s_W^2}\right) d_Q \sqrt{x_q x_Q} f(x_q, x_Q), \tag{5}$$

where $s_W = \sin\theta_W$, $x_a = m_a^2/m_W^2$ and $f(x_q, x_Q)$ stands for the following integral

$$f(x_q, x_Q) = \int_0^1 dx \int_0^{1-x} dy \frac{2-x-y}{1-(1-x_Q)(x+y)-x_q(1-x-y)(x+y)}. \tag{6}$$

Although this integral has analytical solution in the most general case, a compact solution is found in the $x_q = 0$ approximation:

$$f(x_Q) = \frac{1+3x_Q(x_Q-4/3)-2(2x_Q-1)\log(x_Q)}{2(x_Q-1)^3}. \tag{7}$$

We turn now to express the neutron EDM, d_n, in terms of the ones associated with its constituents, u and d. We will use the non-relativistic $SU(6)$ formula, which holds at the electro-weak scale. One then invokes renormalization group to evolve the electric dipole operator down to the hadronic scale. The neutron EDM at the hadronic scale is thus given by

$$d_n = \eta^E \left(\frac{4}{3}d_d - \frac{1}{3}d_u\right) \tag{8}$$

where $\eta^E \sim 0.61$ is the QCD correction factor. We can thus write

$$|d_n| = \left|\eta^E \left(\frac{\alpha}{4\pi s_W^2}\right)\left[\frac{4}{3}\sqrt{x_t\, x_d}|V_{td}|^2 f(x_t) d_t - \frac{1}{3}\sqrt{x_b x_u}|V_{bu}|^2 f(x_b) d_b\right]\right|. \tag{9}$$

For consistency with our analysis, we will use the current quark masses for the u and d quarks. Taking into account that $f(x_t) = 0.22$ and $f(x_b) = 5.07$, numerical evaluation gives:

$$|d_n| = |9.48 \times 10^{-12} d_t - 2.37 \times 10^{-13} d_b| \tag{10}$$

We are now able to constrain the t and b quarks EDMs. The experimental limit on the neutron EDM is given by [3]:

$$d_n^{\mathrm{Exp}} < 2.9 \times 10^{-26} \ \mathrm{e \cdot cm}. \tag{11}$$

As far as the CKM matrix elements is concerned, the latest reported values are [4]:

$$|V_{td}| = (7.40 \pm 0.80) \times 10^{-3}, \tag{12}$$
$$|V_{bu}| = (4.31 \pm 0.30) \times 10^{-3}. \tag{13}$$

To obtain a constraint on the EDM of the t and b quarks, we will assume that either d_t or d_b is nonzero and the other one vanishes. We then obtain

$$|d_t| \; < \; 3.06 \times 10^{-15} \text{ e} \cdot \text{cm}, \tag{14}$$
$$|d_b| \; < \; 1.22 \times 10^{-13} \text{ e} \cdot \text{cm}. \tag{15}$$

It is worth comparing these results with theoretical expectations. The electromagnetic dipolar structure of the top quark has been studied in diverse contexts. We will present a brief survey of the theoretical expectations for the heavy quark EDMs and the corresponding experimental bounds on them. As already mentioned, quark EDM arises in the SM up to the three-loop level [2]. The CP-violating effects arise from the CKM phase. From the estimate for the three-loop contribution for the d quark EDM, one can roughly estimate that d_t is of the order of 10^{-31} to 10^{-32} e-cm. In contrast, in some SM extensions, such as supersymmetric theories and multi-Higgs doublet models (MHDMs) the situation is quite different as fermion EDM can arise at the one-loop level [5, 6, 7, 8, 9, 10], with the corresponding estimate for d_t being about ten orders of magnitude larger than the SM prediction. It has been noted that this result opens the window for experimental detection [11]. Such large values for the t quark EDM are due to the fact that in models with an extended Higgs sector the fermion EDM scales as m_f^3. Heavy quark EDM has been calculated extensively in the framework of MHDMs, where CP violation can arise due to the scalar exchange between quarks. It has been found that [6], assuming that the dominant contribution arises from the lightest neutral Higgs boson h, d_t can be of the order of 10^{-19} e-cm for $m_h = 100$ GeV. On the other hand, CP violation can also arise in the neutral Higgs sector of a two-Higgs doublet model (THDM) if there is a phase in the Higgs-fermion-fermion interactions. This scenario has been explored in Ref [7]. Although this analysis refers to the type II THDM, it is also valid for type I and III THDMs after some replacements of the coupling constants. For particular values of the parameters of THDM with CP violation in the neutral scalar sector, the t quark EDM can reach values of the order of 10^{-18}-10^{-19} for $m_h = 100 - 300$ GeV [11]. Even more, in models with two or three Higgs doublets, CP violation can also arise in the charged Higgs sector. Assuming $m_{H^\pm} = 200$ GeV, authors of Ref. [8] showed that d_t is of the order of 10^{-22}. As for supersymmetric theories, t quark EDM can arise at the one-loop level in the minimal supersymmetric standard model (MSSM) even without generation mixing. The CP-violating phase is provided by the chargino and neutralino mixing matrices as well as the squarks $q_L - q_R$ mixing matrices. In this model the d_t can receive gluino, chargino and neutralino contributions [9]. For convenient values of the parameters involved in the calculation, the t quark EDM is typically of the order of 10^{-19}-10^{-20} e-cm, which is much smaller than the values expected in the Higgs sector. The EDM of heavy quarks is also sensitive to non-universal extra dimensions [10]. It has been estimated that the t quark EDM can reach values as high as 10^{-20} e-cm for a value of the compactification scale of 300 GeV. A more suppressed value for d_t, of the order of 10^{-22} e-cm, was obtained recently from the one–loop contribution of an anomalous tbW vertex including both left– and right–handed complex components [12], and more recently the one–loop contribution of an anomalous CP–odd $WW\gamma$ vertex [13] was used to estimate the value of 10^{-21} e-cm for d_t.

On the experimental side, the data on the rare flavor changing neutral current decay $b \to s\gamma$ has been used to analyze potential top–mediated new physics effects [14, 15]. In Ref. [14] the contribution to the $b \to s\gamma$ decay from both the magnetic and electric dipole moments of the top quark was studied. Those authors report an upper bound on d_t of the order of 10^{-16} e-cm, which is stronger than that obtained here. Detailed studies have also been made to probe the structure of the $tt\gamma$ vertex at future e^-e^+, pp, $p\bar{p}$, $\gamma\gamma$ and $\mu^-\mu^+$ colliders. Although there is no doubt that it is of extreme importance the measurement of the static EDM of the t quark, due to its short lifetime it will be much less difficult to measure the t quark nonstatic EDM. One of the main tasks of the future linear e^-e^+ collider will be to determine the top quark properties, mainly via $t\bar{t}$ production. It has been found [16] that it will be possible to determine values of the order of $d_t \sim 10^{-17}$ e-cm with 10^4 $t\bar{t}$ events in a linear collider running at c.m. energies of $\sqrt{s} = 500$ GeV. The t quark dipole moment could also be probed at a future photon collider via $\gamma\gamma \to t\bar{t}$ [17]. It has been found that for c.m. energies of 500 GeV, a photon collider would be sensitive to values of the order of $d_t \sim 10^{-17}$ e-cm. Therefore, the limits on the t quark EDM that would be obtained at a photon collider are of the same order than those that would be obtained at an e^-e^+ linear collider.

As far as the b quark EDM is concerned, values which differ by one or two orders of magnitude than those obtained for the t quark EDM have been reported in the literature. To our knowledge there is not yet any upper bound reported in the literature, but there are estimates derived from multi–Higgs models. In this class of theories, the estimates for d_b are reported to lie in the range of $10^{-23} - 10^{-22}$ e-cm, whereas the estimate 10^{-23} e-cm was derived from the one–loop contribution of a CP–odd $WW\gamma$ vertex [13].

In conclusion, we have used the current experimental limit on the neutron EDM to derive upper bounds on the t and b quarks EDMs. Our constraint on d_t is weaker than that derived from the $b \to s\gamma$ decay [14]. Roughly speaking, the bounds on d_t and d_b are at least four orders of magnitude above the predictions obtained in several specific models incorporating new sources of CP violation. It means that there is a potential window to explore CP–violating effects in the third quark family.

Acknowledgments

We acknowledge financial support from CONACYT (México).

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